中国科学技术大学
University of Science and Technology of China

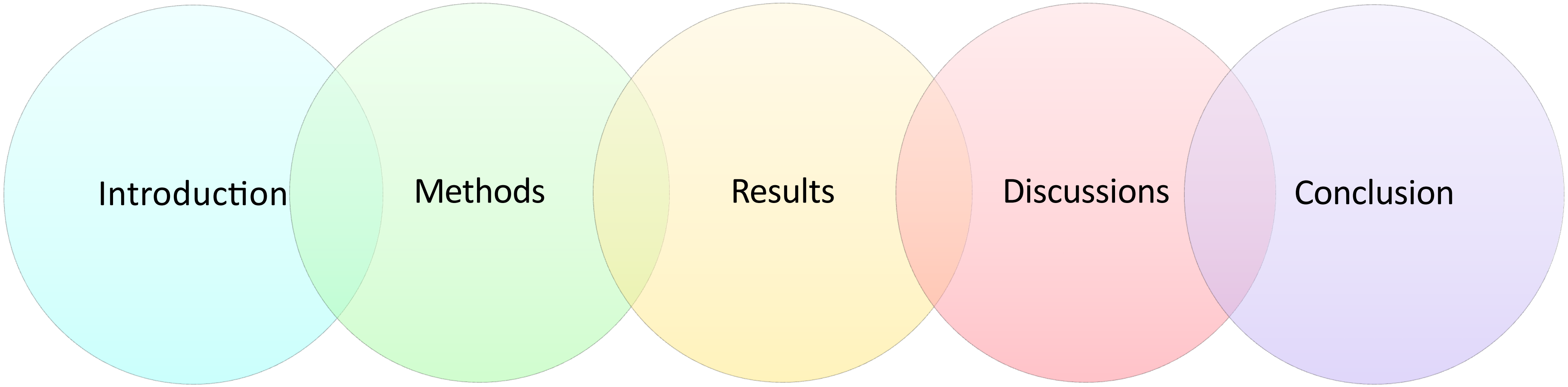
Possibility for Electric Conductors with Like Charges to Attract

Theoretical Analysis & Numerical Stimulation based on Mathematica

Siwei Luo; Can’en Yang

25 June 2024

Table of Contents

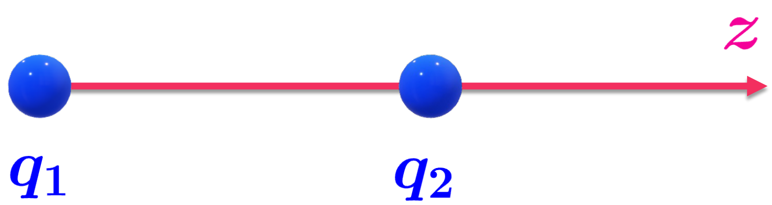


Section I

Introduction

Two Point Charges

$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} \longrightarrow \boldsymbol{F} := \boldsymbol{F}_{21}$$



Coulomb's Law

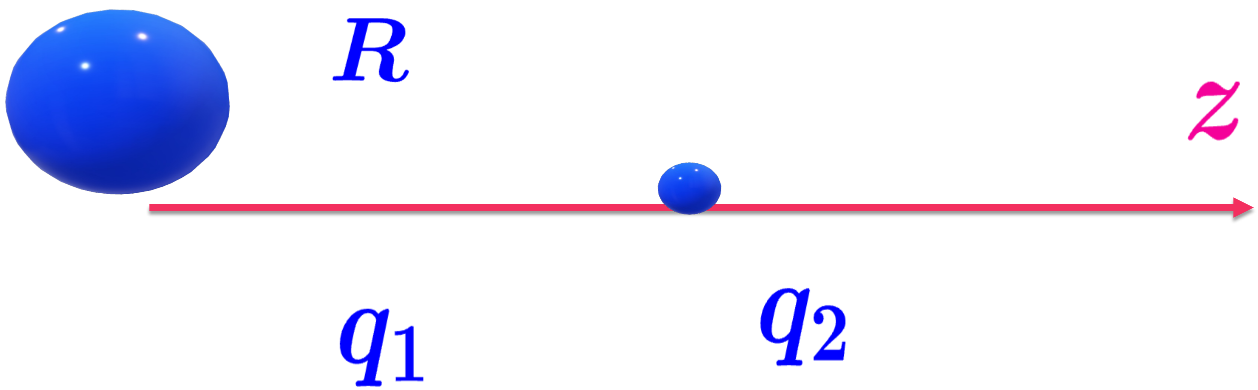
$$\boldsymbol{F} = -\frac{q_1 q_2 \hat{\boldsymbol{z}}}{4\pi\varepsilon_0 z^2}$$

$$q_1 q_2 > 0$$

$$-\frac{q_1 q_2}{4\pi\varepsilon_0 z^2} < 0$$

IMPOSSIBLE ✗

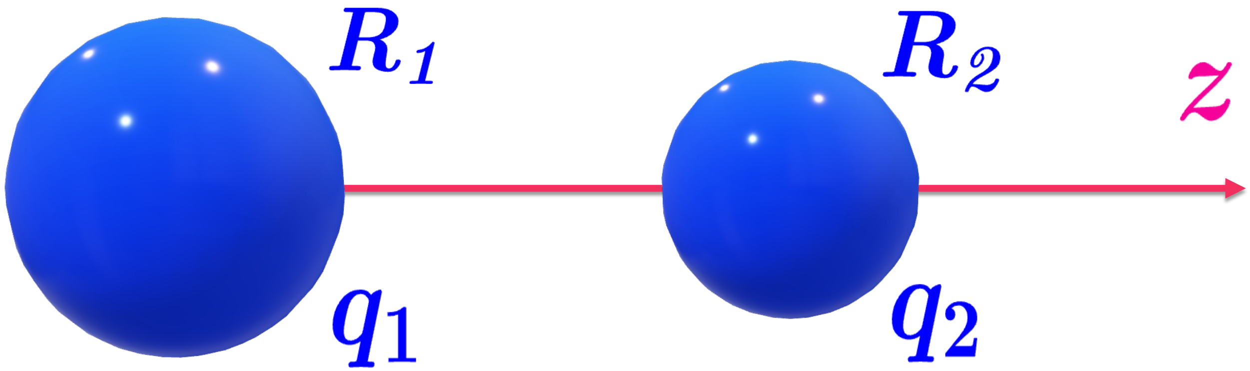
One Point Charge & One Sphere Conductor



$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} \Longrightarrow \boldsymbol{F} := \boldsymbol{F}_{21}$$

$$\boxed{\boldsymbol{F} = ?}$$

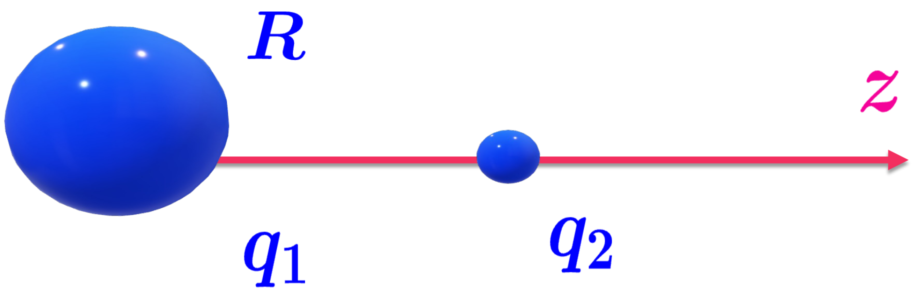
Two Sphere Conductors

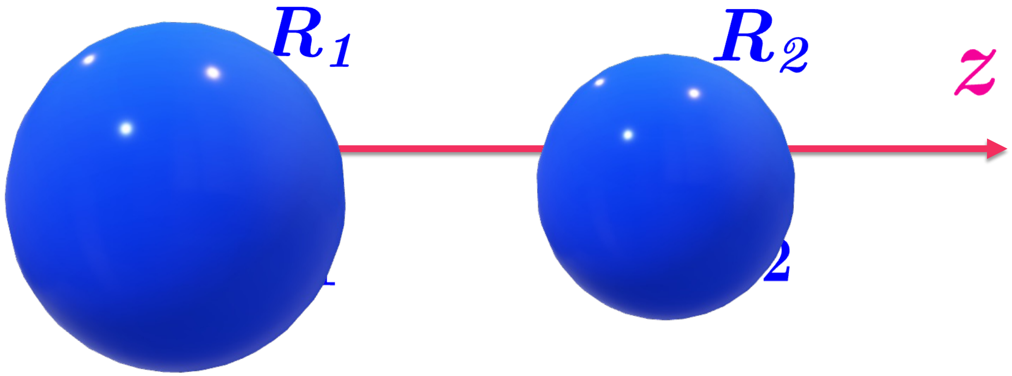


$$\boldsymbol{F}_{12} = -\boldsymbol{F}_{21} \Longrightarrow \boldsymbol{F} := \boldsymbol{F}_{21}$$

$$\boxed{\boldsymbol{F} = ?}$$

Key Problem between Two Conditions Remaining





$F = ?$

How to figure out F ?

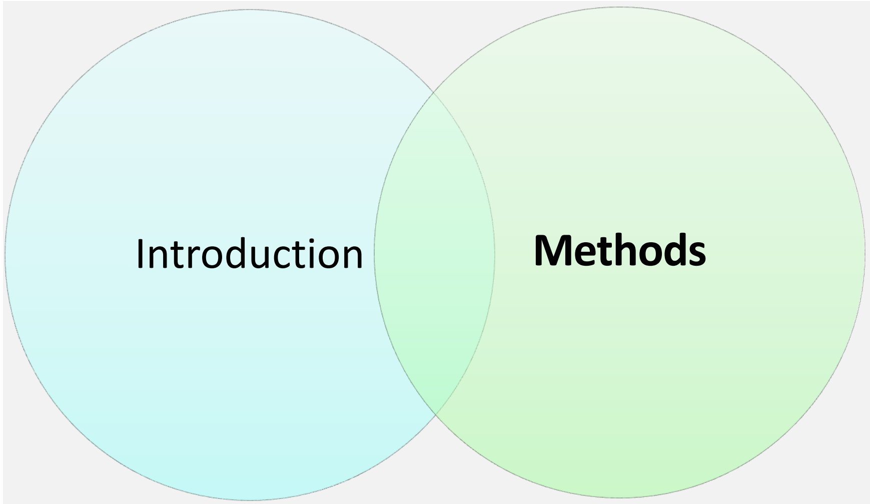
Judge the direction of F

Or

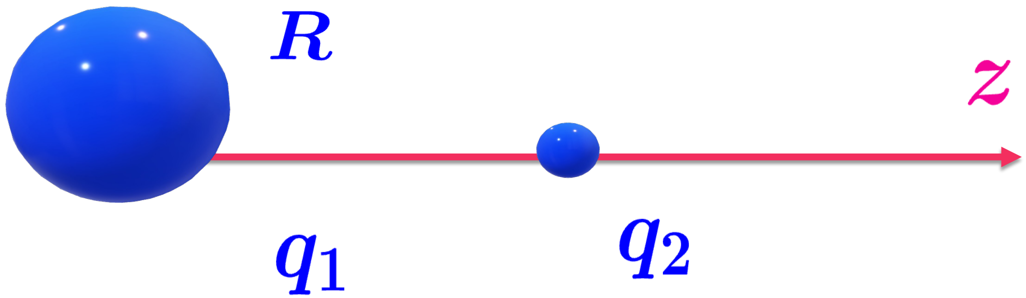
Theory

Stimulation

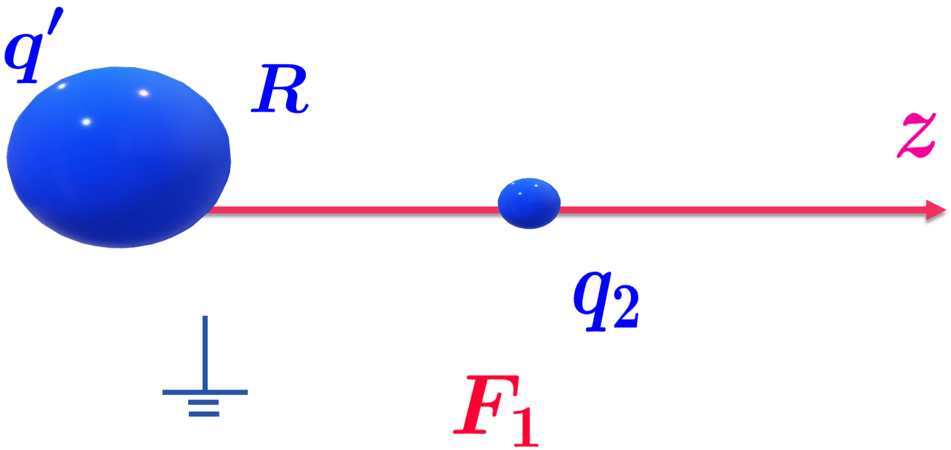
Section II

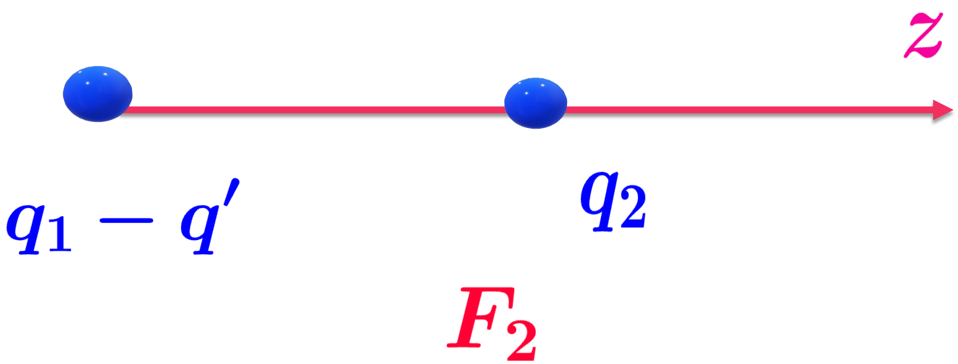


One Point Charge & One Sphere Conductor

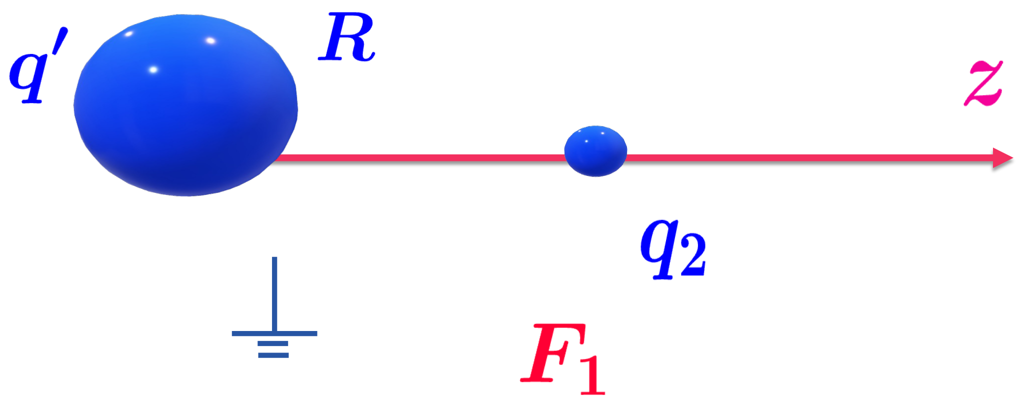


$$F = F_1 + F_2$$

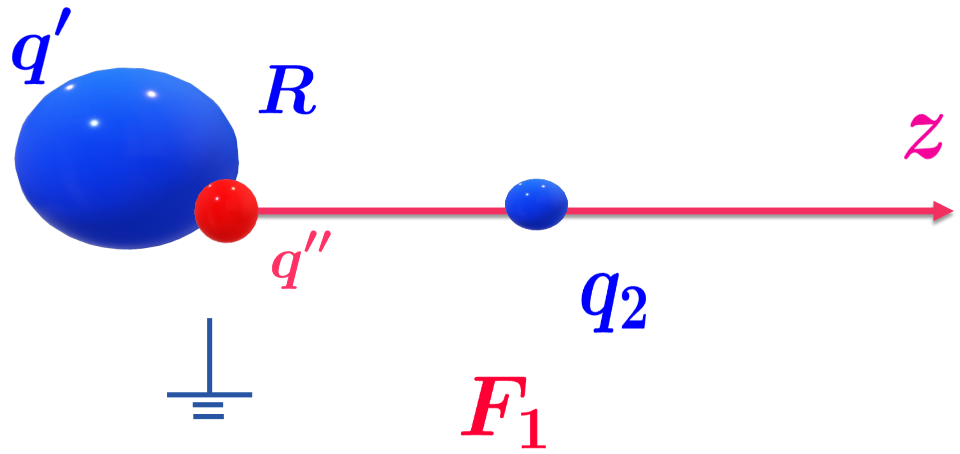




One Point Charge & One Sphere Conductor

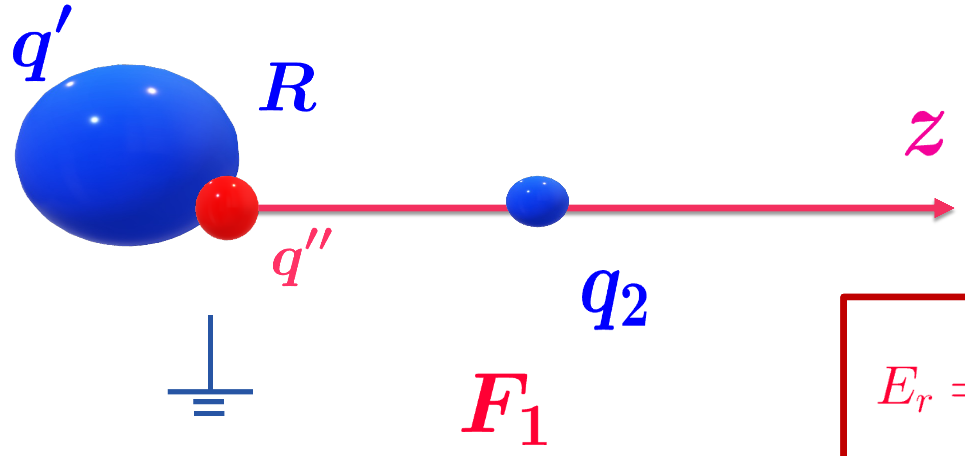


$$U\,\big|_{\partial q_1} = 0$$



$$z' = \frac{R^2}{z} \qquad q' = -\frac{R}{z}q_2$$

One Point Charge & One Sphere Conductor



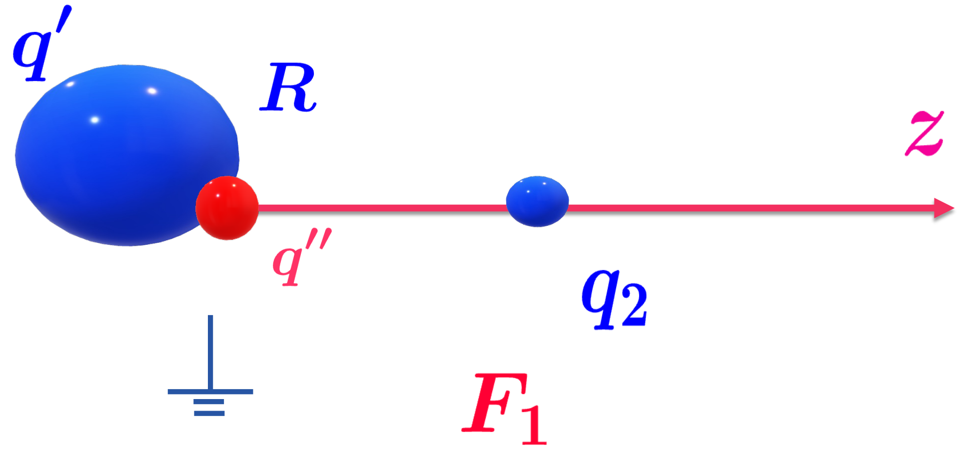
$$U = \frac{q_2}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{R}{zR_2}\right)$$

$$E_r = -\frac{\partial U}{\partial r} = \frac{q_2}{4\pi\varepsilon_0}\left[\frac{r - z\cos\theta}{R_1^3} - \frac{R(r - z'\cos\theta)}{zR_2^3}\right]$$

$$z' = \frac{R^2}{z} \qquad q' = -\frac{R}{z}q_2$$

$$\sigma_e = \varepsilon_0 E_r\,|_{r=R} = \frac{q_2(R - z^2/R)}{4\pi(R^2 + z^2 - 2Rz\cos\theta)^{\frac{3}{2}}}$$

One Point Charge & One Sphere Conductor

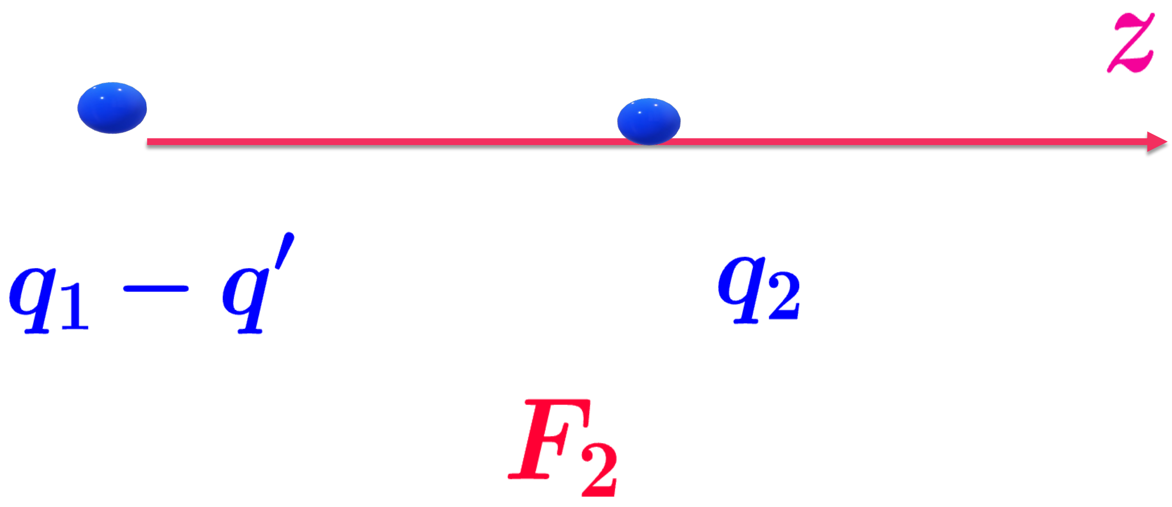


$$
\begin{aligned}
q'' &= \oiint_{Q_1} \sigma_e dS, \\
&= \oiint_{Q_1} -\frac{Q_2}{4\pi}\frac{d^2/R - R}{(R^2+d^2-2Rd\cos\theta)^{\frac{3}{2}}} R d\theta d\varphi, \\
&= -\frac{Q_2(d^2-R^2)}{4\pi}\int_0^{2\pi} d\varphi \int_0^{\pi} \frac{d\theta}{(R^2+d^2-2Rd\cos\theta)^{\frac{3}{2}}}, \\
&= -\frac{R}{d}Q_2 = q'.
\end{aligned}
$$

$$\sigma_e = \varepsilon_0 E_r \,|_{r=R} = \frac{q_2(R - z^2/R)}{4\pi(R^2+z^2-2Rz\cos\theta)^{\frac{3}{2}}}$$

$$\boldsymbol{F}_1 = -\frac{Rzq_2^2\hat{\boldsymbol{z}}}{4\pi\varepsilon_0(z^2-R^2)^2}$$

One Point Charge & One Sphere Conductor



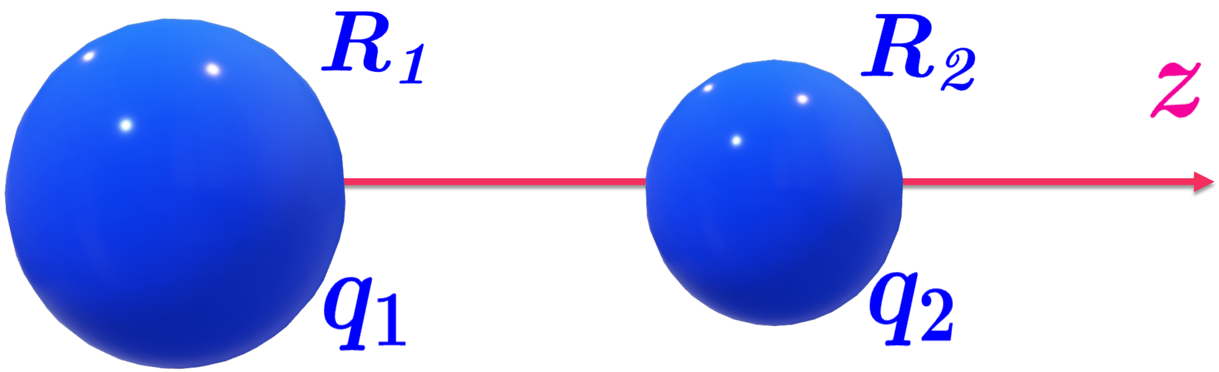
$$q'' = q'$$

$$\boldsymbol{F}_2 = \frac{q_2(q_1 - q')\hat{\boldsymbol{z}}}{4\pi\varepsilon_0 z^2}$$

$$\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$$

$$\boldsymbol{F} = -\frac{q_2\hat{\boldsymbol{z}}}{4\pi\varepsilon_0}\left[\frac{q_2 R z}{\left(z^2 - R^2\right)^2} - \frac{q_1 z + q_2 R}{z^3}\right]$$

Two Sphere Conductors



Stimulation by Mathematica

Constructing Electrical Images

Set $\mathfrak{M}$ $\longrightarrow$ Set M

Construction Method

$\boldsymbol{r}_1 = (0,0,0)$

$\boldsymbol{r}_2 = (0,0,z)$

$$\text{Set } \mathfrak{M} \xrightarrow[k \to +\infty]{\mathfrak{M}_k = \{\boldsymbol{r}_k, q_k\}} \text{Set } M$$

$k \geq 3$:

$\mathfrak{M}_k \in q_1$

Add image of $\mathfrak{M}_k$ under q_2

Add opposite of image to center of q_2

$\mathfrak{M}_k \in q_2$

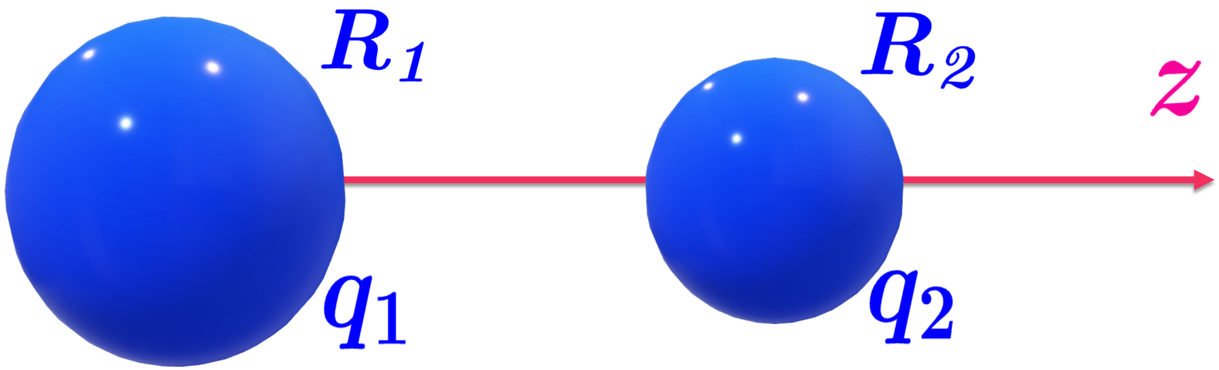
Add image of $\mathfrak{M}_k$ under q_1

Add opposite of image to center of q_1

$-\lg \frac{q_n}{q_1} > 3$ Stop !!!

Calculation Method

Stimulation by Mathematica



$$\mathfrak{M}_k = \{\boldsymbol{r}_k, q_k\}$$

$$\boldsymbol{E}_t = \boldsymbol{E} - \boldsymbol{E}_0$$

$$U(\boldsymbol{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0 ||\boldsymbol{r} - \boldsymbol{r}_i||}$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E}_t dS = \oiint_{Q_1} \sigma \left(\boldsymbol{E} - \frac{\sigma}{2\varepsilon_0}\boldsymbol{n}\right) dS.$$

$$\sigma = \varepsilon_0 E_r \,|_{r=R} = -\varepsilon_0 \frac{\partial U}{\partial r}$$

$$\boldsymbol{E} = -\nabla U$$

Calculation Method

Stimulation by Mathematica

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \left(\boldsymbol{E} - \frac{\sigma}{2\varepsilon_0} \boldsymbol{n} \right) dS,$$

$$= \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS.$$

$$\sigma = \varepsilon_0 E_r \mid_{r=R} = -\varepsilon_0 \frac{\partial U}{\partial r}$$

$$\oiint_{Q_1} \sigma \boldsymbol{E} dS = \hat{\boldsymbol{z}} \oiint_{Q_1} \sigma (E_r \cos\theta - E_\theta \sin\theta) dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta.$$

$$\oiint_{Q_2} \sigma^2 \cos\theta dS = R_1^2 \int_0^{2\pi} d\varphi \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta,$$
$$= 2\pi R_1^2 \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta,$$

Calculation Method

Stimulation by Mathematica

$$\oiint_{Q_1} \sigma \boldsymbol{E} dS = \hat{\boldsymbol{z}} \oiint_{Q_1} \sigma (E_r \cos\theta - E_\theta \sin\theta) dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta.$$

$$\oiint_{Q_2} \sigma^2 \cos\theta dS = R_1^2 \int_0^{2\pi} d\varphi \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta,$$
$$= 2\pi R_1^2 \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta,$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

Integral → **Riemann Sum**

Edit & Layout Siwei Luo

Section III

One Point Charge & One Sphere Conductor

$$\boldsymbol{F} = -\frac{q_2 \hat{\boldsymbol{z}}}{4\pi\varepsilon_0}\left[\frac{q_2 R z}{(z^2 - R^2)^2} - \frac{q_1 z + q_2 R}{z^3}\right]$$

$$q_1 = q_2 = 1\ \text{C},\ R = 1\ \text{m}$$

Attract

$$f(z) := \frac{q_2 R z}{(z^2 - R^2)^2} - \frac{q_1 z + q_2 R}{z^3} > 0$$

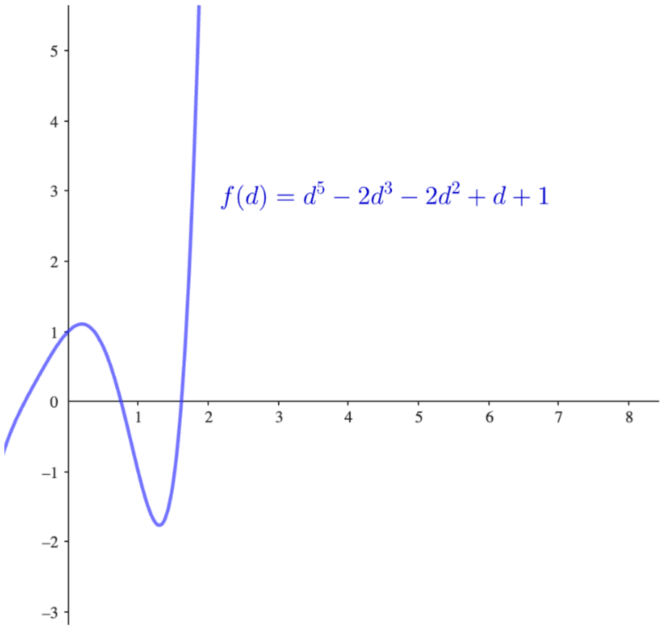
$$f(z) = z^5 - 2z^3 - 2z^2 + z + 1$$

One Point Charge & One Sphere Conductor

$$q_1 = q_2 = 1 \text{ C}, \ R = 1 \text{ m}$$

Attract

$$f(z) = z^5 - 2z^3 - 2z^2 + z + 1 > 0$$

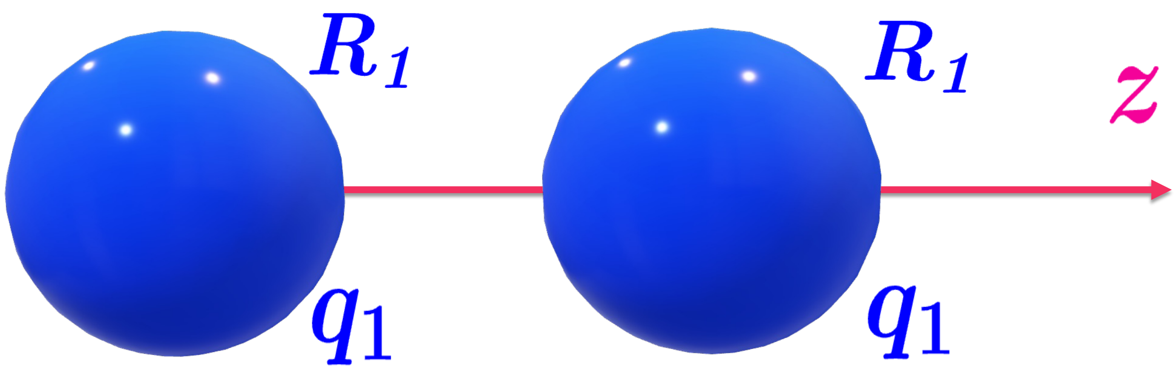


$$f(z) := \frac{q_2 R z}{(z^2 - R^2)^2} - \frac{q_1 z + q_2 R}{z^3} > 0$$

Possible to Attract!

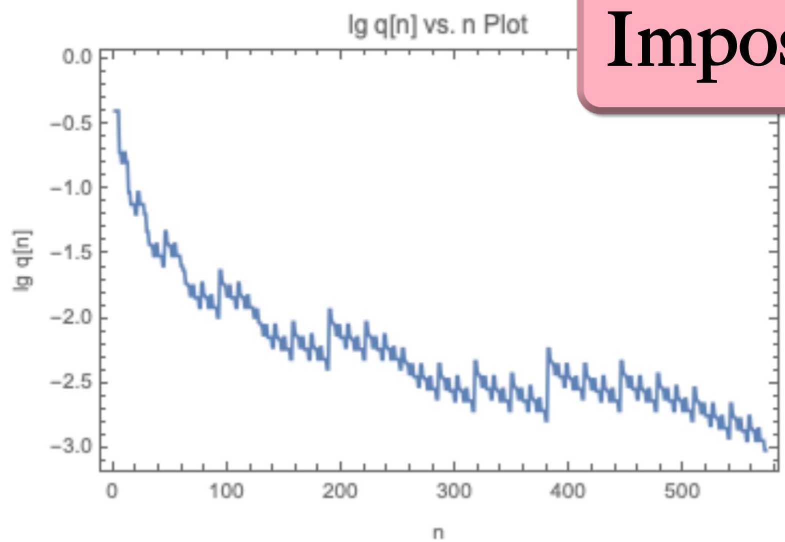
Two Sphere Conductors

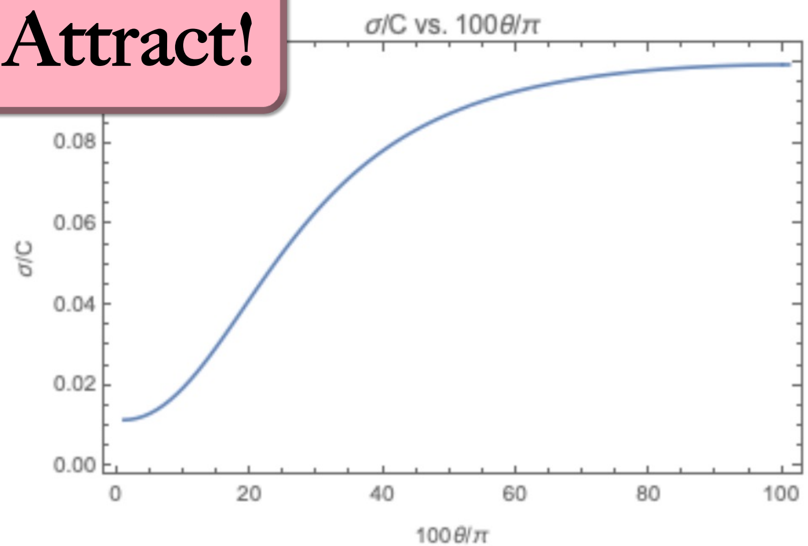
Stimulation by Mathematica



$$q_1 = q_2 = 1 \text{ C},$$
$$R_1 = R_2 = 1 \text{ m},$$
$$z = 2.5 \text{ m}.$$

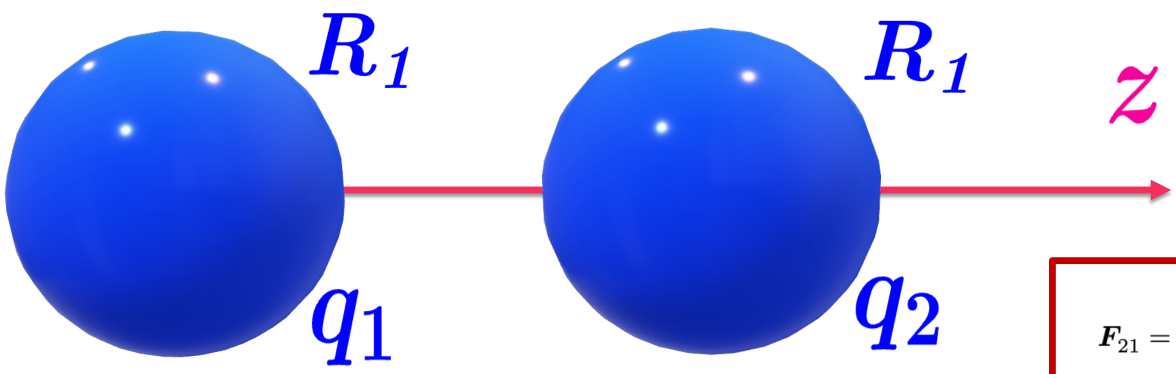
Impossible to Attract!





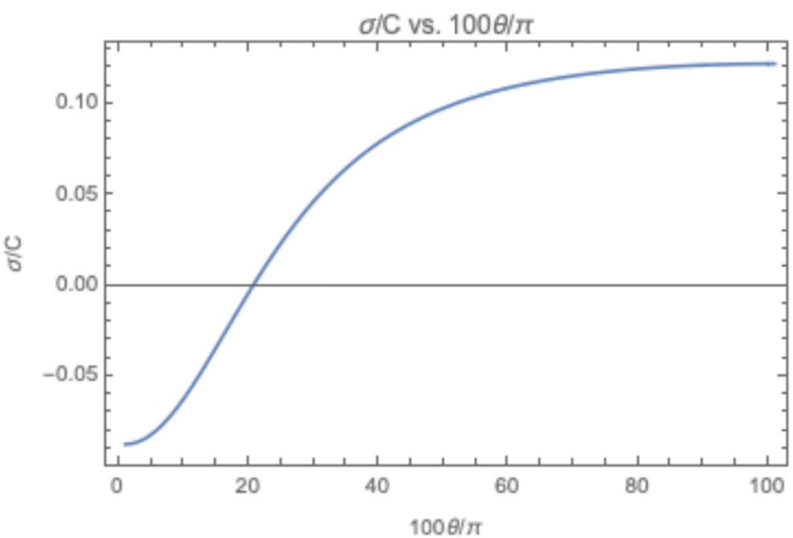
Two Sphere Conductors

Stimulation by Mathematica



$$2q_1 = q_2 = 2\ \text{C},$$
$$R_1 = R_2 = 1\ \text{m},$$
$$z = 2.5\ \text{m}.$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^{\pi} \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^{\pi} \sigma^2 \cos\theta \sin\theta d\theta.$$

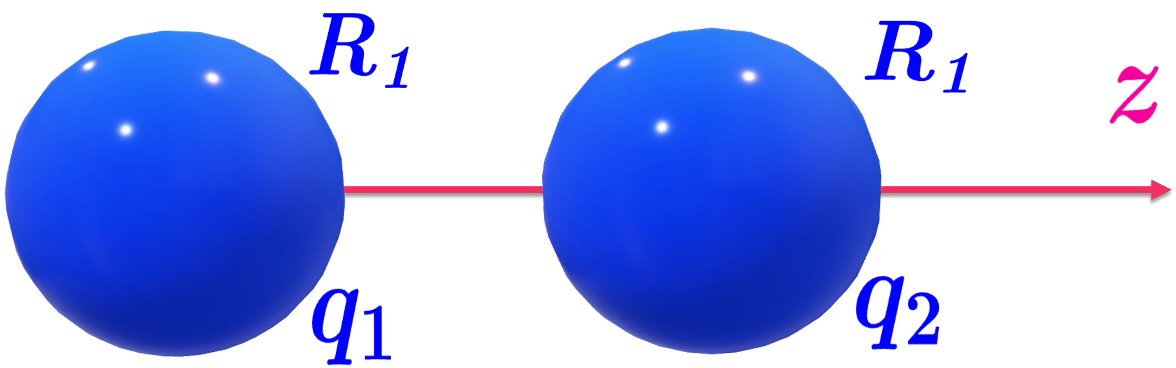


$$F = -1.91327 \times 10^9\ \text{N}$$

Impossible to Attract!

Two Sphere Conductors

Stimulation by Mathematica



$$10q_1 = q_2 = 10 \text{ C},$$
$$R_1 = R_2 = 1 \text{ m},$$
$$z = 2.5 \text{ m}.$$

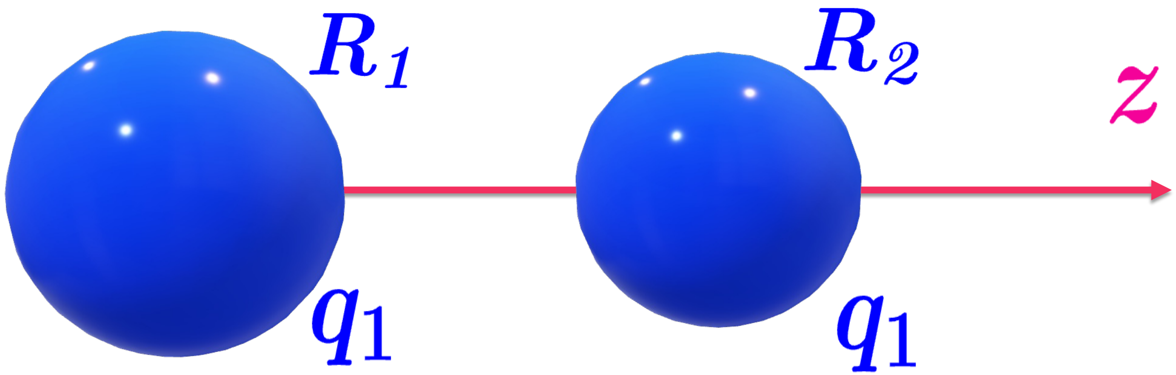
$$F = +1.08622 \times 10^{10} \text{ N}.$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^{\pi} \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^{\pi} \sigma^2 \cos\theta \sin\theta d\theta.$$

Possible to Attract!

Two Sphere Conductors

Stimulation by Mathematica



$$q_1 = q_2 = 1 \text{ C},$$
$$R_1 = 1.25 R_2 = 1 \text{ m},$$
$$z = 2.3 \text{ m}.$$

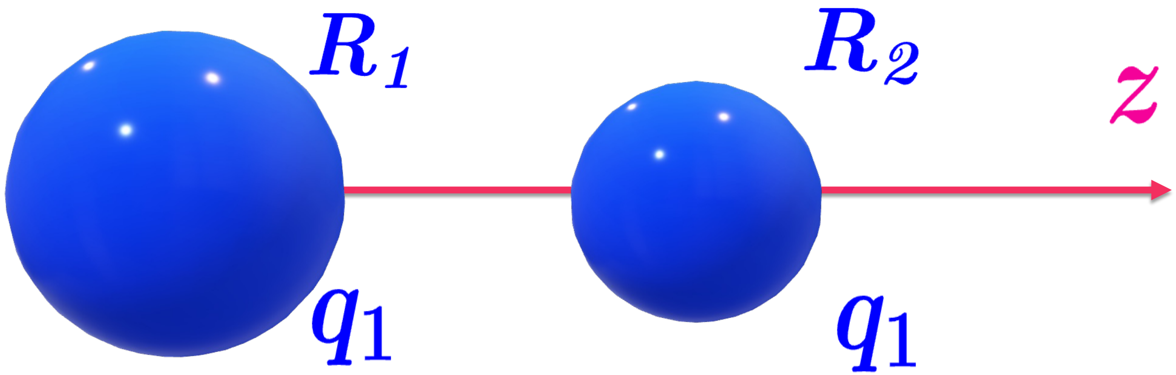
$$F = -1.27076 \times 10^9 \text{ N}$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

Impossible to Attract!

Two Sphere Conductors

Stimulation by Mathematica



$$q_1 = q_2 = 1 \text{ C},$$
$$R_1 = 2R_2 = 1 \text{ m},$$
$$z = 2.0 \text{ m}.$$

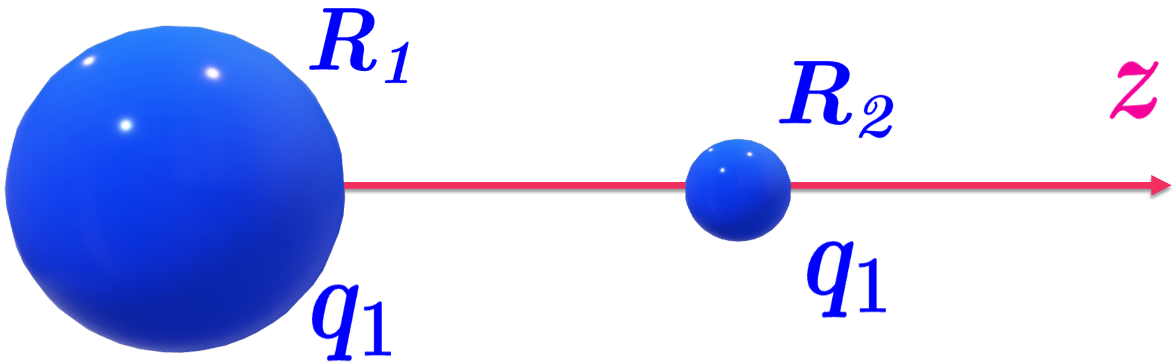
$$F = -1.38513 \times 10^9 \text{ N}$$

$$\boldsymbol{F}_{21} = \oint\!\!\!\oint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oint\!\!\!\oint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

Impossible to Attract!

Two Sphere Conductors

Stimulation by Mathematica



$$q_1 = q_2 = 1 \text{ C},$$
$$R_1 = 10R_2 = 1 \text{ m},$$
$$z = 1.6 \text{ m}.$$

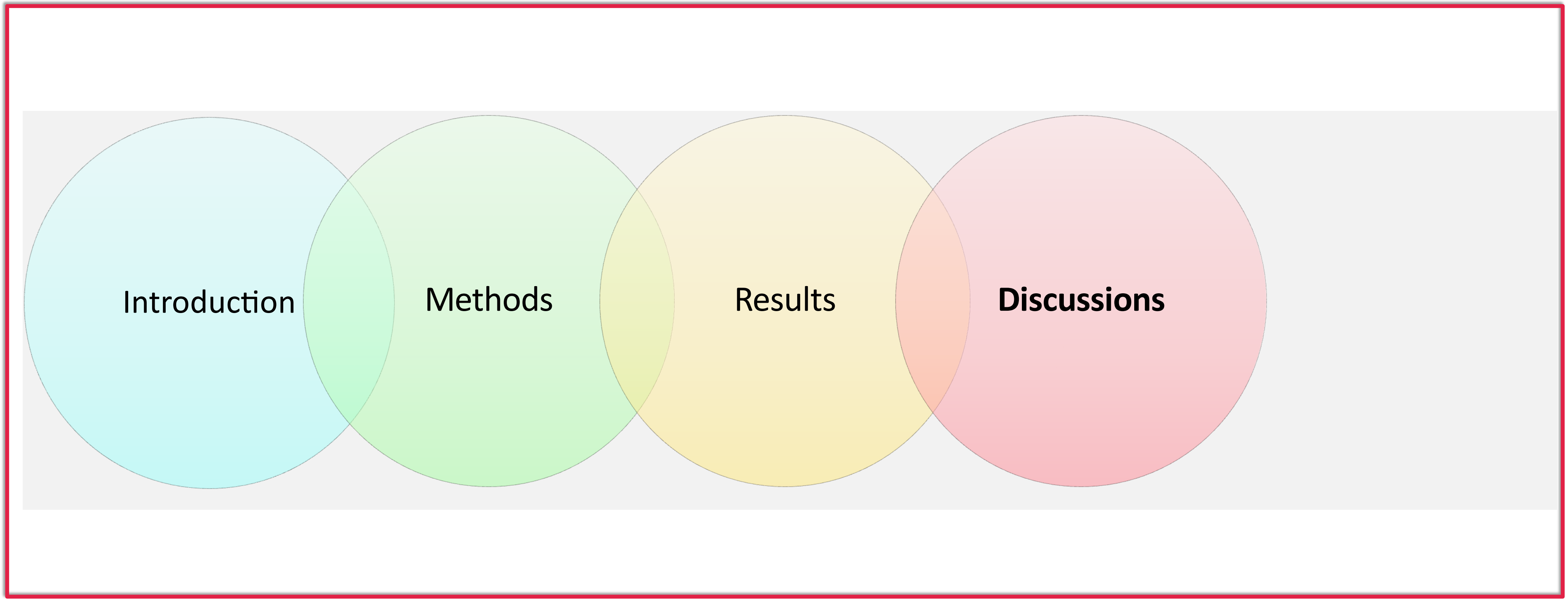
$$F = +2.04922 \times 10^8 \text{ N}.$$

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$
$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

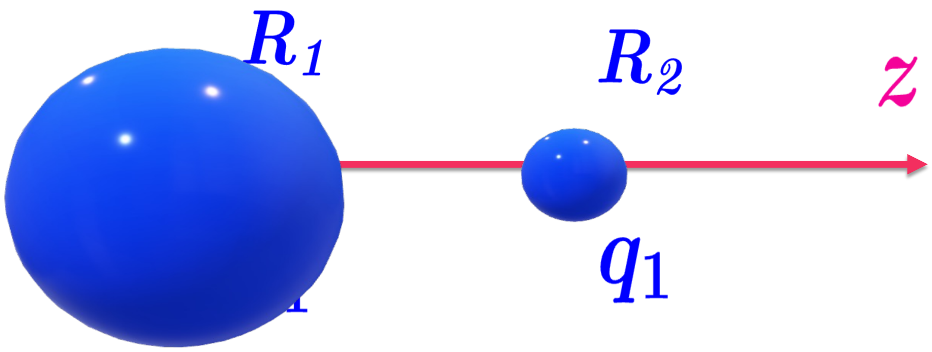
Possible to Attract!

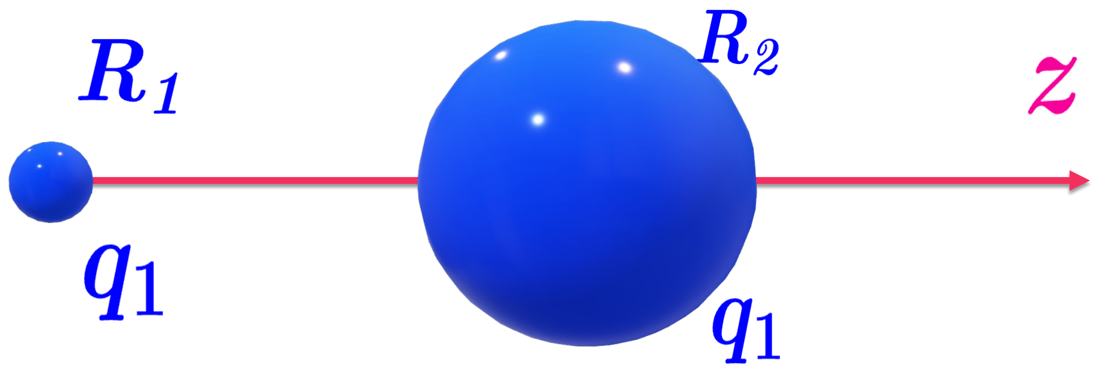
Edition & Layout Siwei Luo

Section IV



Newton's Law III ?





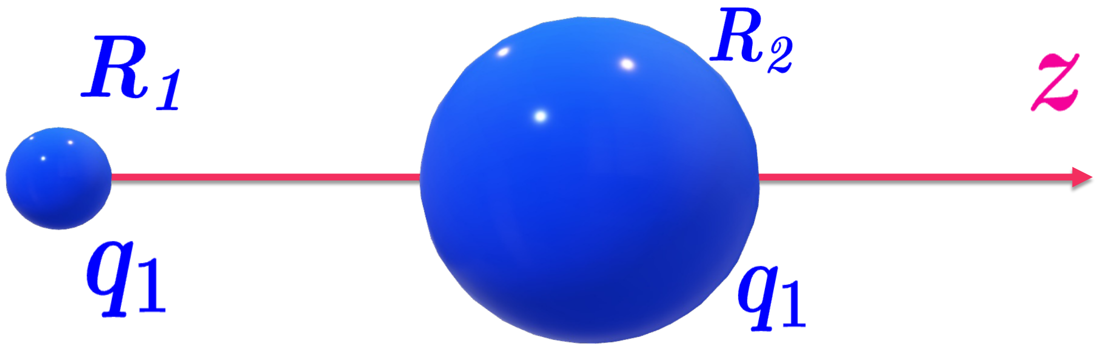
$$q_1 = q_2 = 1 \text{ C},$$
$$R_1 = 10R_2 = 1 \text{ m},$$
$$z = 1.6 \text{ m}.$$

$$q_1 = q_2 = 1 \text{ C},$$
$$10R_1 = R_2 = 1 \text{ m},$$
$$z = 1.6 \text{ m}.$$

$$F = +2.04922 \times 10^8 \text{ N}.$$

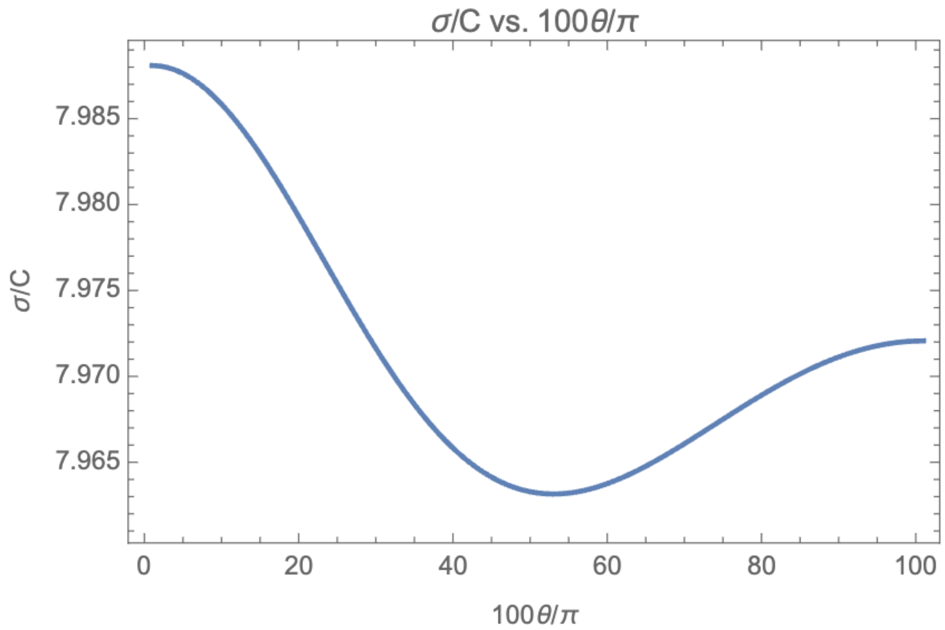
$$F' = +2.11589 \times 10^8 \text{ N}$$

Sigma-Theta Diagram



$$q_1 = q_2 = 1\ \text{C},$$
$$10R_1 = R_2 = 1\ \text{m},$$
$$z = 1.6\ \text{m}.$$

$$F' = +2.11589 \times 10^8\ \text{N}$$



Calculation Improvement ?

Theory

Stimulation by Mathematica

$$E_\theta << E_r$$

$$U\ |_{\partial q_1} = const$$

$$\boldsymbol{F}_{21} = \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

$$E_\theta \longrightarrow 0$$

Calculation Improvement ?

$$\boldsymbol{F}_{21} = \oiint_{Q_1} \sigma \boldsymbol{E} dS - \frac{\hat{\boldsymbol{z}}}{2\varepsilon_0} \oiint_{Q_2} \sigma^2 \cos\theta dS,$$

$$= 2\pi R_1^2 \hat{\boldsymbol{z}} \int_0^\pi \sigma \sin\theta (E_r \cos\theta - E_\theta \sin\theta) d\theta - \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

$$\boldsymbol{E_\theta \longrightarrow 0}$$

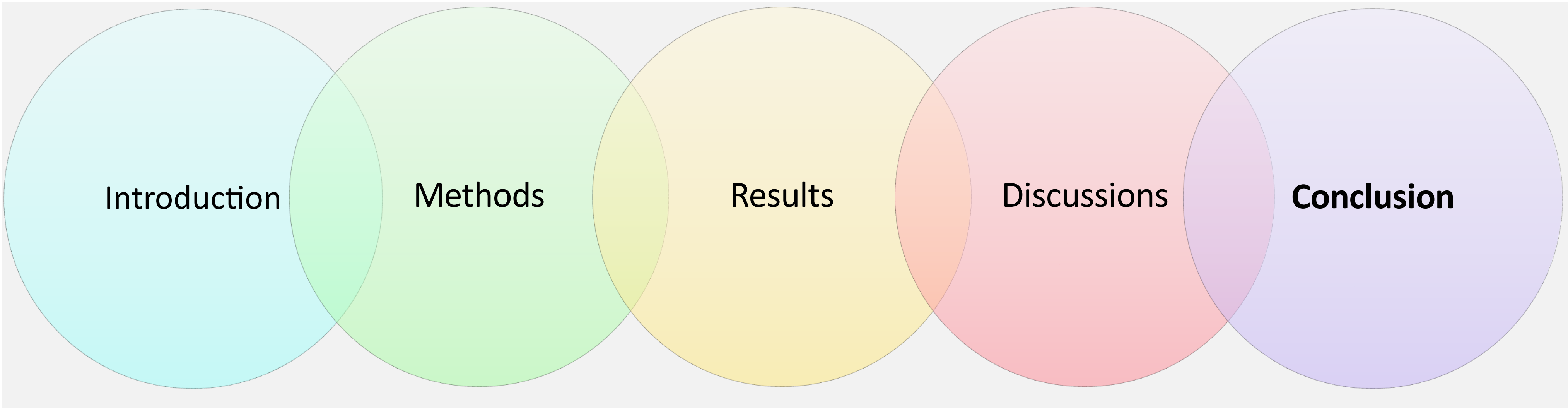
$$\boldsymbol{F}_{21} = \frac{\pi R_1^2 \hat{\boldsymbol{z}}}{\varepsilon_0} \int_0^\pi \sigma^2 \cos\theta \sin\theta d\theta.$$

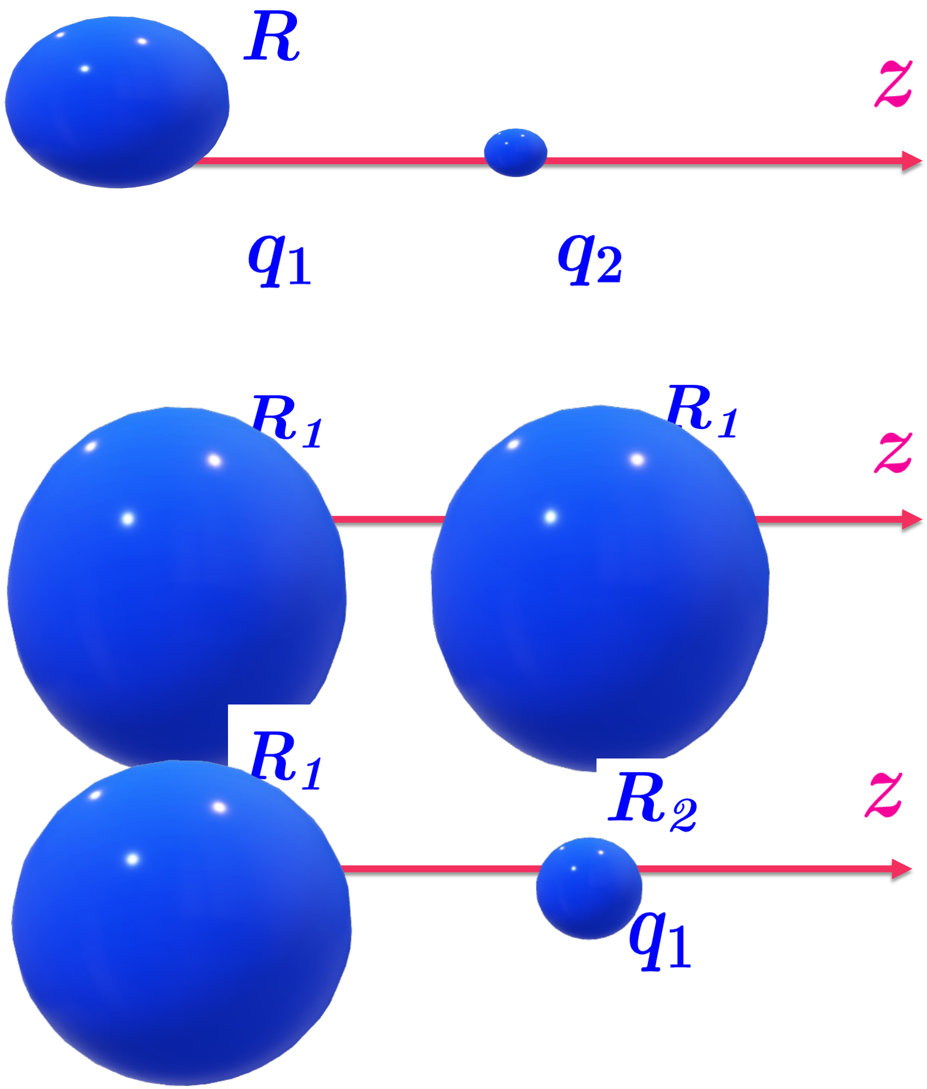
Improvement ?

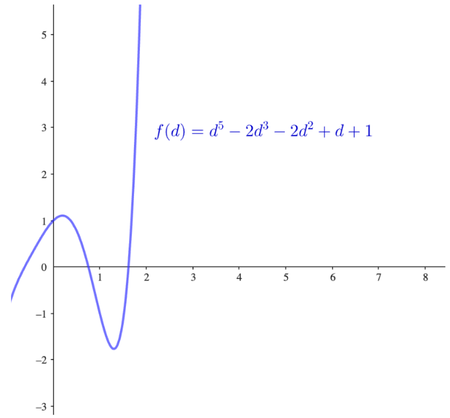
Maybe NOT!

Edition & Layout Siwei Luo

Section V



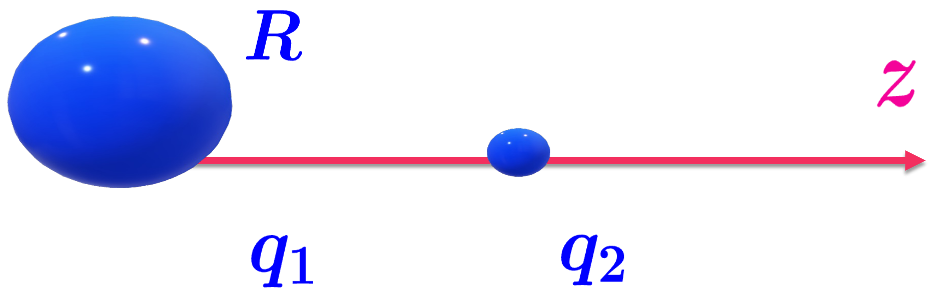


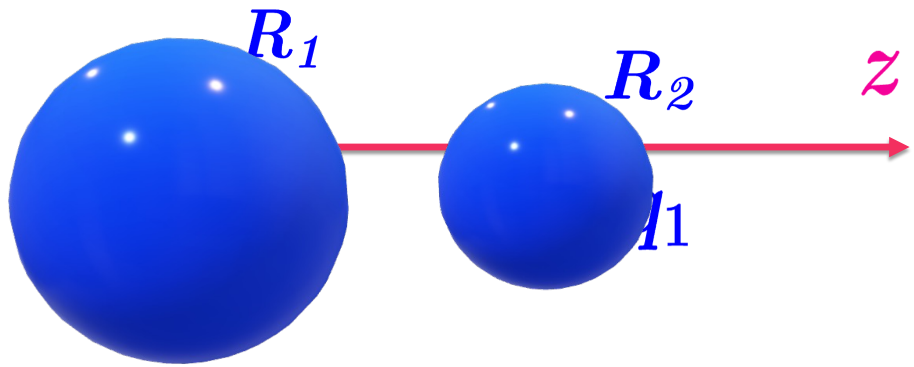


$$F = +1.08622 \times 10^{10}\ \text{N}.$$

$$F = +2.04922 \times 10^{8}\ \text{N}.$$

Possible to Attract!





Possibility for Electric Conductors with Like Charges to Attract

Theoretical Analysis & Numerical Stimulation based on Mathematica

University of Science and Technology of China

Thank you!

Siwei Luo; Can'en Yang

25 June 2024